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Boundary Layer Separation and Reattachment[1]

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A review of recent developments in a new model for boundary-layer separation will be presented. The separation model encompasses time-varying as well as steady-flow conditions. Engineering criteria in terms of calculable pressure gradients are obtained for both laminar and turbulent boundary-layer separation. These criteria, together with limited information on time-dependent separations, should be of value in predicting flows in turbomachinery. The reattachment of boundary layers is also shown to fit within the separation model.

Boundary-layer separation is a limiting factor in the performance of nearly all fluid-flow devices. In turbomachinery, turbulent separation is encountered in the diffuser flow, and laminar separation can be encountered on the compressor and turbine blades. The problem of boundary-layer separation in turbomachinery can be further complicated due to unsteady or time-varying flow conditions. It is readily apparent that an understanding of boundary-layer separation can be applied directly to improvement of the operation of several components in turbomachines.

Prandtl (ref. 1) first advanced a model of a zero wall-shear stress, laminar, separation boundary layer. Prandtl's model is adequate for an ideal flow in which the boundary layer has sufficient time to adjust to the separation conditions. However, for turbulent boundary layers, and also for laminar boundary layers where separation occurs rapidly, the layer cannot adjust to the ideal conditions required by Prandtl.

Sandborn and Kline (ref. 2) proposed a boundary-layer separation model which incorporated the Prandtl "steady separation" and also an "unsteady separation." This model suggested a separation region in which the boundary layer "transitioned" to the separation conditions. The recent experimental measurements by Liu (refs. 3 and 4) for turbulent

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boundary-layer separation confirmed the general concept of the transition region. It has, however, become increasingly apparent that the concept of "steady" and "unsteady" separation, suggested by Sandborn and Kline, is not an adequate description for all boundary-layer separations.

The present paper presents a more general model for boundary-layer separation. The new model is based on the concept of an adjustment time for the boundary layer. It is now possible to include time-dependent boundary layers in the model. Based on these new concepts, an engineering criterion for a wide range of boundary-layer separation is constructed.

BOUNDARY-LAYER SEPARATION MODEL

The definition of boundary-layer separation is usually associated with zero wall shear stress or in terms of a limiting streamline (ref. 5). Such a definition, while exact, appears too idealized for general engineering applications. Separation must be defined mainly as an unwanted viscous region, where mass and heat transfer at the surface are greatly reduced.

A general definition of boundary layer separation is *the removal of viscous restraints at the wall.* This definition is intended to imply that the wall shear has a negligible effect on the further development of the flow characteristics in the separation region. It does not necessarily require that the wall shear vanish. The wall shear may be positive, negative, or zero, as long as it is small compared to inertia terms. The definition is necessarily vague in order to encompass both time-average and time-dependent boundary-layer evaluations.

The flow visualization studies of Kline (ref. 6) were instrumental in showing that turbulent separation began as an unsteady, three-dimensional phenomenon. The separation was seen to develop into what was thought to be a statistically "steady" type of separation. Thus, the Sandborn-Kline model assumed that a transition region existed where the flow changed from the "unsteady" to the "steady" type separation. Liu (ref. 4) was able to experimentally demonstrate the existence of the transition region. However, it was readily apparent that conditions were certainly far from steady at any point in the transition process.

It is now realized that the original Sandborn-Kline model is too restricted. The transition region should be viewed as a region over which the flow adjusts to the removal of viscous restraints at the wall. Figure 1 is a sketch of the separation transition region. The extent of the region depends directly on the body geometry and the free-stream conditions. At the start of the region, boundary conditions are such that viscous effects at the surface are negligible. At the end of the region, the complete velocity distribution has adjusted to the surface condition. If a laminar boundary layer approaches separation in a sufficiently mild, adverse,

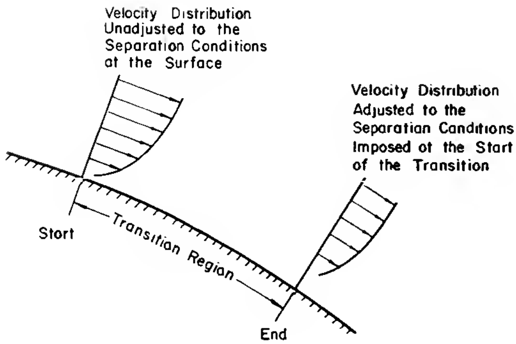


FIGURE 1.—*Sketch of the proposed separation model.*

pressure-gradient flow, the whole layer can adjust to the boundary conditions. Thus, for some laminar separation cases the transition region reduces to a point. On the other hand, if the approach to separation is very rapid, the velocity distribution cannot adjust to the wall conditions and a transition region exists. The nature of the turbulent boundary layer is such that a transition region would be expected.

The adjustment time required depends on how quickly the outer flow in the boundary layer comes to equilibrium with the separation boundary condition. For a turbulent boundary layer, it is well known that the outer part of the layer is inertia dominated and responds very slowly to changes in the inner region of the boundary layer. It is possible for the inner region to "so to speak" vanish completely, with only a secondary effect on the outer region. Thus, the adjustment time of a turbulent boundary layer to a viscous condition at the surface is quite long. The laminar boundary layer is much thinner, so it can adjust to boundary conditions at the surface in a shorter time.

Measurements for turbulent boundary layers at the start of the separation transition region indicate a small positive mean wall-shear stress (ref. 4). Only at the end of the transition region does the mean wall-shear stress appear to be zero. This residual shear is thought to be due to the unsteady nature of the flow. The description of "steady" separation for the end of the transition region, employed by Sandborn and Kline, now appears to be misleading. For the time-varying free-stream flows it is found that the wall-shear stress may vary from zero to a negative value as the velocity profile adjusts. Thus, specification of the wall-shear stress in the separation region is quite subjective. It appears that the wall-shear stress may be of only secondary importance in defining the separation region.

The empirical velocity profile parameter correlations developed by Sandborn and Kline (ref. 2) are employed to identify the beginning and end of the separation transition region. Figure 2 is a plot of the velocity profile form factor versus the ratio of displacement to boundary-layer

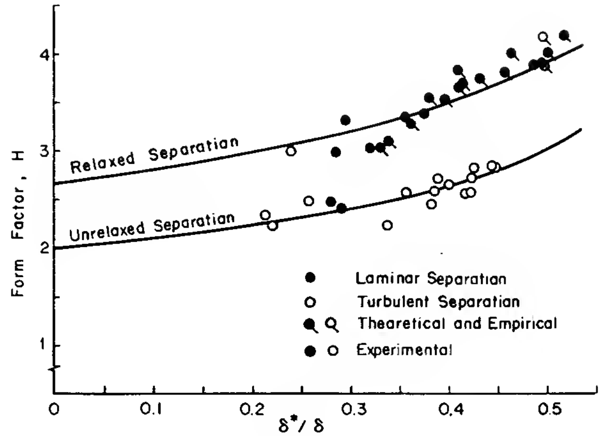


FIGURE 2.—*Boundary-layer separation correlations.*

thickness. The empirical correlation curves identified as "relaxed" and "unrelaxed" separation are those originally given by Sandborn and Kline and identified as "steady" and "unsteady" separation. The words "relaxed" and "unrelaxed" are presently thought to be a better technical description of the boundary layer than "adjusted" and "unadjusted". All available identified separation velocity profiles are included in figure 2. The profiles are identified according to whether they are laminar or turbulent boundary layers. Points are also identified as either experimental or analytical.

In general, laminar boundary-layer separations are found to be near the relaxed separation limit. This is in keeping with the classical separation model of Prandtl. The turbulent boundary-layer separations are likely to be identified with the unrelaxed separation limit. The obvious exceptions are

(1) Liu's measured turbulent boundary layer profile at the point of relaxed separation ($H=3.0$, $\delta^*/\delta=0.24$) (ref. 7)

(2) Coles' "law" of the wake turbulent separation profile ($H=4.1$, $\delta^*/\delta=0.5$) (ref. 8)

(3) Two measured laminar separation bubble profiles ($H=2.49$, $\delta^*/\delta=0.286$ and $H=2.48$, $\delta^*/\delta=0.282$) (ref. 9)

The concept of boundary-layer adjustment distance implies that partially relaxed separations should also be found. It is suspected that the two laminar separation bubble measurements, shown in figure 2 at $H=3.05$, $\delta^*/\delta=0.325$ and $H=3.00$, $\delta^*/\delta=0.290$ may indicate partially relaxed separation. The analytical points at $H=3.10$, $\delta^*/\delta=0.341$ and $H=3.05$, $\delta^*/\delta=0.332$ computed by Liu (ref. 7) may also represent partially relaxed separation. Evidence of partially relaxed separation will

also be demonstrated in the sections on time varying flows and reattachment of boundary layers.

The present model of boundary-layer separation assumes that the velocity distributions for laminar and turbulent flows are similar. The analysis of Sandborn and Liu (ref. 4) demonstrated that the similarity at the relaxed separation location could be justified by considering the equations of motion. At the location of relaxed separation, the equations reduced to the inertia terms equal to the pressure force over the major part of the layer. The shear forces were only important in matching the pressure force at the surface. For unrelaxed separation, it does not appear possible to make a specific statement about the magnitudes of the inertia and shear terms. The magnitudes will depend on how rapidly the separation is approached. Thus, the shear terms may range in importance from that of a flat plate flow to negligible values compared to the inertia terms.

ENGINEERING CRITERIA FOR BOUNDARY-LAYER SEPARATION

The correlations shown in figure 2 imply that one-parameter families of velocity profiles exist at separation. This model may prove too simple to cover all possible separation cases; the time-varying flow cases may be exceptions. However, as a first approximation it is possible to construct engineering criteria based on the results of figure 2. The three parameters employed in figure 2 are not readily computed in boundary-layer prediction methods. Thus, it is necessary to recast the correlations in terms of a calculable pressure-gradient parameter. The analysis of Sandborn and Kline (ref. 2) led to a criterion for laminar relaxed separation in terms of the pressure-gradient parameter

$$\lambda_\delta \equiv \frac{\delta^2}{\nu} \frac{dU_1}{dx}$$

The recent analysis of Liu and Sandborn (ref. 3) produced a criterion for laminar separation in terms of the velocity profile form factor, H, versus the pressure-gradient parameter

$$\lambda_\theta \equiv \frac{\theta^2}{\nu} \frac{dU_1}{dx}$$

Figure 3 shows the criterion given by Liu and Sandborn. Experimental and analytical data points are included in figure 3. The laminar curve is shown dashed out to include the turbulent relaxed separation point measured by Liu (ref. 4). It appears reasonable to identify the laminar criterion with relaxed separation. Table I lists the coordinates for the curves of figure 3.

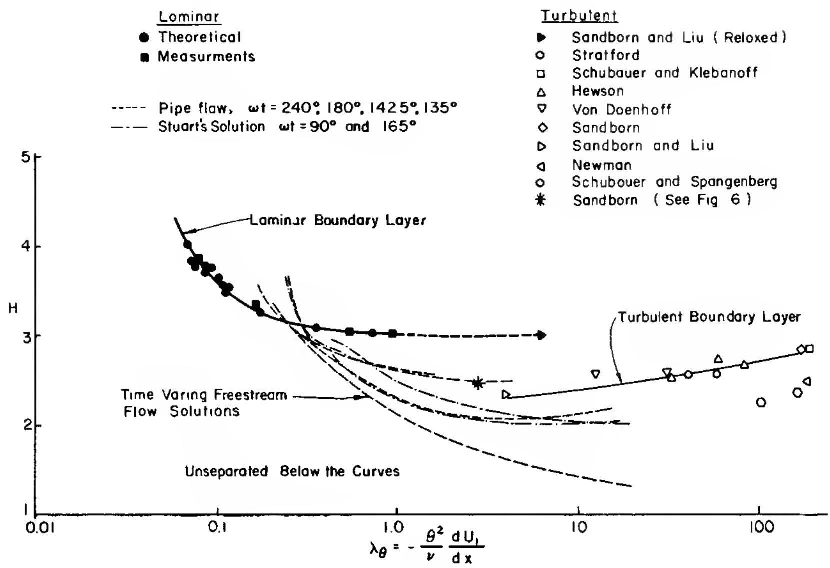


FIGURE 3.—*Boundary separation criteria.*

The pressure-gradient parameter, λ_θ, can be computed for laminar boundary layers, once the external velocity distribution is specified, by employing Thwaites' relation (ref. 10) for momentum thickness:

$$\theta^2 = 0.45\nu U_1^{-6} \int_0^x U_1^5\, dx \qquad (1)$$

This relation was demonstrated by Liu and Sandborn to be an accurate prediction of θ for the analytical laminar boundary-layer solutions included in figure 3. The x-location of the separation can be estimated from the simple relation given by Stratford (ref. 11)

$$x_s^2 C_p \left(\frac{dC_p}{dx}\right)^2 = \text{constant} \qquad (2)$$

Stratford gave the constant as 7.64×10^{-3}. Curle and Skan (ref. 12) suggested a value of 1.04×10^{-2} for the constant. The recent calculations by Liu and Sandborn indicate that a constant midway between the constant of Stratford and that of Curle and Skan may be the best. Thus, the separation criterion of figure 3 is a quick means of evaluating H or δ^* at separation, once θ and x_s are computed from equations (1) and (2).

An engineering criterion for the unrelaxed separation case may also be expected in terms of H and λ_θ. Included in figure 3 is a plot of unrelaxed turbulent boundary-layer separation profile parameters. Much of this

TABLE I.—*Separation Criteria*

Laminar separation				Turbulent separation[1]			
Curve		Plotted Points		Curve		Plotted Points	
λ_θ	H	λ_θ	H	λ_θ	H	λ_θ	H
.06	4.29			4.0	2.30		
.08	3.86	0.007	4.03	5.0	2.32	3.95	2.36
.10	3.62	0.0725	3.83	6.0	2.34	12.6	2.59
.12	3.51	0.0750	3.8	7.0	2.36	31.5	2.58
.14	3.40	0.08	3.80	8.0	2.375	32.5	2.50
.20	3.22	0.08	3.88	9.0	2.385	43.0	2.57
.30	3.12	0.09	3.79	10	2.40	58.5	2.56
.40	3.10	0.096	3.78	12	2.42	59	2.74
.50	3.08	0.092	3.75	15	2.45	83	2.66
.60	3.07	0.0875	3.73	20	2.485	170	2.83
.70	3.06	0.105	3.67	25	2.515	180	2.46
.80	3.05	0.108	3.60	30	2.54	192	2.84
.90	3.04	0.115	3.54	35	2.56	102	2.25
1.00	3.03	0.120	3.535	40	2.58	167	2.35
		0.117	3.51	50	2.61		
		0.167	3.36	60	2.635		
		0.178	3.26	70	2.66		
		0.360	3.10	80	2.675		
		0.545	3.05	90	2.69		
		0.725	3.05	100	2.71		
		0.9425	3.01	120	2.74		
				150	2.77		
				200	2.82		
				250	2.86		

[1] Approximate formula (straight line) $y = 2.12 + 0.306 \log_{10}x$.

data is now available in tabular form (ref. 13), so that parameters such as λ_θ can be determined in a uniform manner.

The curve through the turbulent data in figure 3 was drawn to agree in placement with the way in which the data points lie about the unrelaxed separation curve in figure 2. The coordinates of this unrelaxed separation curve are listed in table I.

The variation of H with λ_θ for the unrelaxed turbulent separation is opposite to that found for laminar flow. This trend of H increasing with λ_θ is not understood at present. Turbulent separation measurements are always subject to question due to secondary-flow effects. However, secondary flow acts to make θ larger than it would be in two-dimensional

flow. Corrections for secondary flow would be expected to reduce the range of λ_θ.

The "partially relaxed" curves shown in figure 3 will be discussed in the section on time-varying flows. Figure 4 demonstrates that the unrelaxed separation correlation also may be applied to axisymmetric boundary layers. The axisymmetric profile parameters are defined (ref. 14) as

$$\left.\begin{array}{ll} \text{Outside Curvature} & \text{Inside Curvature} \\ \delta^*_{3-d}=\dfrac{1}{R}\displaystyle\int_R^{R+\delta}\left(1-\frac{U}{U_1}\right)r\,dr & \delta^*_{3-d}=\dfrac{1}{R}\displaystyle\int_{R-\delta}^{R}\left(1-\frac{U}{U_1}\right)r\,dr \\ \theta_{3-d}=\dfrac{1}{R}\displaystyle\int_R^{R+\delta}\frac{U}{U_1}\left(1-\frac{U}{U_1}\right)r\,dr & \theta_{3-d}=\dfrac{1}{R}\displaystyle\int_{R-\delta}^{R}\frac{U}{U_1}\left(1-\frac{U}{U_1}\right)r\,dr \end{array}\right\} \quad (3)$$

a.—*Relaxed.*

b.—*Unrelaxed.*

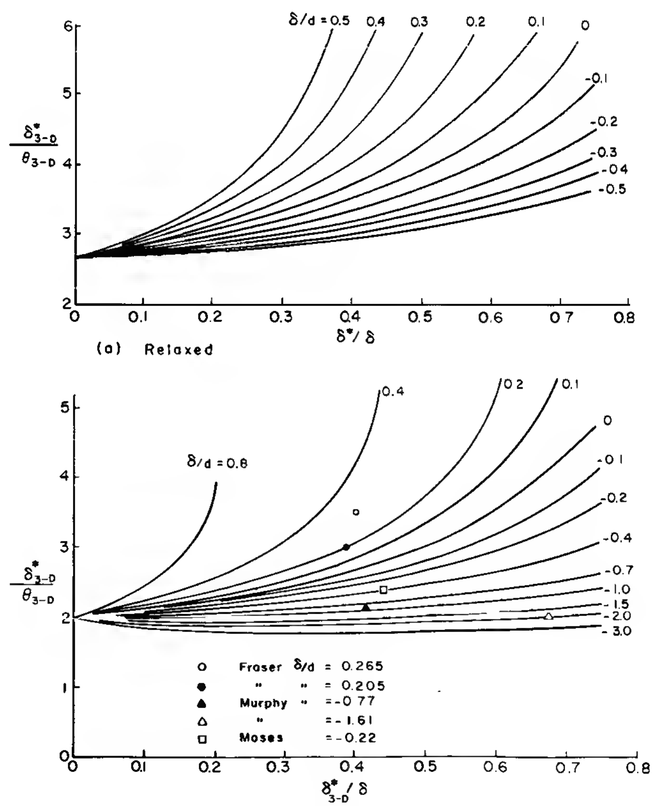


FIGURE 4.—*Axisymmetric boundary-layer separation correlation.*

where d is the diameter and R is the radius of the body at the point of separation. In figure 4, positive values of δ/d are for internal conduits, such as diffusers, and negative values of δ/d are for external curvature, such as cones or cylinders. The data of Murphy (ref. 15), shown in figure 4, are for ogive cylinders. For the ogives, the local curvature at the point of separation was employed to specify d. Correlations of H_{s-d} in terms of λ_{θ_s-d} have not been determined, since only limited measurements are available.

Although a criterion for turbulent separation is given in figure 3, it is of limited value at present. The difficulty encountered is that predictions of the boundary layer do not agree with the experimental data near separation. Figure 5 compares the calculations presented at the Stanford Conference on Computations of Turbulent Boundary Layers (Vol. I of ref. 13) for the Schubauer-Klebanoff flow. The measured separation point and separation criterion are also noted in figure 5. In no case do the predictions appear to approach the separation criterion. Prediction of the Schubauer-Spangenberg and Newman separation points, shown in figure 3, is somewhat better. Unfortunately, these results do not appear to belong with the other measured separation flows.

TIME-VARYING FLOW

An important problem in boundary-layer separation is that of a time-varying free-stream flow. This type of flow is characteristic of that encountered by blades in turbomachines. Analytical treatment of this type of flow is extremely limited. The relaxation or adjustment time

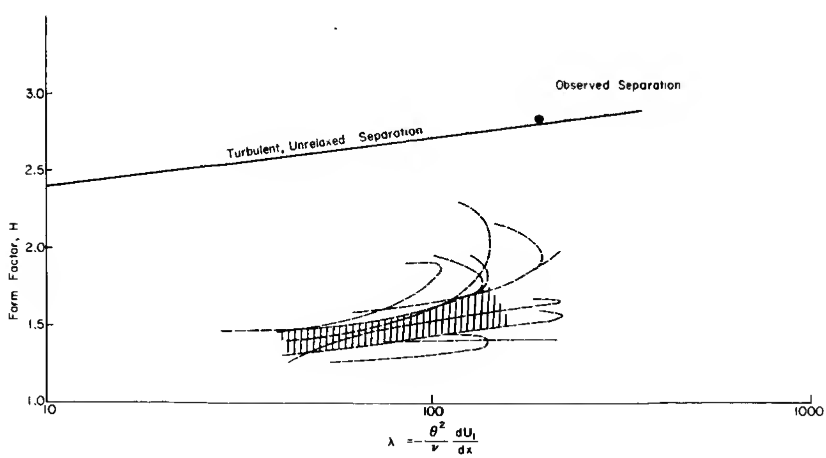


FIGURE 5.—*Comparison of turbulent separation criterion with boundary-layer predictions of the Schubauer-Klebanoff flow (ref. 13).*

concept for boundary-layer separation makes it possible to take time-varying flows into account. The degree to which a boundary layer can adjust to the boundary conditions will depend on the frequency and amplitude of the free-stream velocity. This section examines both turbulent and laminar flows with varying free-stream velocities. The specific point where $\tau_w(t)=0$ is evaluated with respect to the separation criteria. This zero wall-shear point is only part of the general separation model; however, it does give some insight into conditions for these special flows.

Measurements in a turbulent boundary layer with a pulsing free-stream flow were made by the author and C. Feiler at the NASA Lewis Research Laboratory. The physical setup and heat transfer measurements for this flow were reported by Feiler (ref. 16). A siren was employed to produce a pulsing free-stream velocity, such as that shown in figure 6*a*. The free-stream velocity was of the order of 6 feet per second in order to produce an approximate pure velocity pulse. The hot-wire anemometers were employed to measure the boundary-layer velocity distribution along a flat plate. One wire was fixed in the free stream and the second wire was traversed through the layer. At each height, an oscilloscope trace was recorded to obtain the instantaneous variation in wire voltage. Typical hot-wire traces are shown in figure 6*a*. From these traces, it was possible to compute the boundary-layer velocity at each instance in the pulse. Complete boundary-layer velocity profiles are computed in this manner. The faired boundary-layer velocity distributions are shown in figure 6*b*.

The measurements, while subject to considerable scatter, give an accurate picture of a transient turbulent boundary layer approaching separation. Figure 6*c* is a plot of the velocity profile form factor H versus δ^*/δ for boundary-layer development from "reattachment" to "separation." The point D represents the instant of time when it appeared the flow very near the surface reversed direction or stopped. The point F indicates where the flow near the surface appeared to again reverse direction or stop. Thus, the "separation" and "reattachment" points are taken as points where $\tau_w(t)\simeq 0$. The faired velocity profiles at "separation" and "reattachment" were found to fall on the unrelaxed separation correlation curve. These results are very much in keeping with the concept that the boundary layer is unable to adjust to the separation condition. The relation

$$\frac{\theta}{\delta}=\frac{\delta^*}{\delta}-\frac{9}{5}\left(\frac{\delta^*}{\delta}\right)^2 \tag{4}$$

very closely fits the parameter development to the separation region. This relation comes from a simple velocity profile approximation (ref. 17)

$$\frac{U}{U_1}=1-\zeta(t)\left(1-\frac{y}{\delta}\right)^m \tag{5}$$

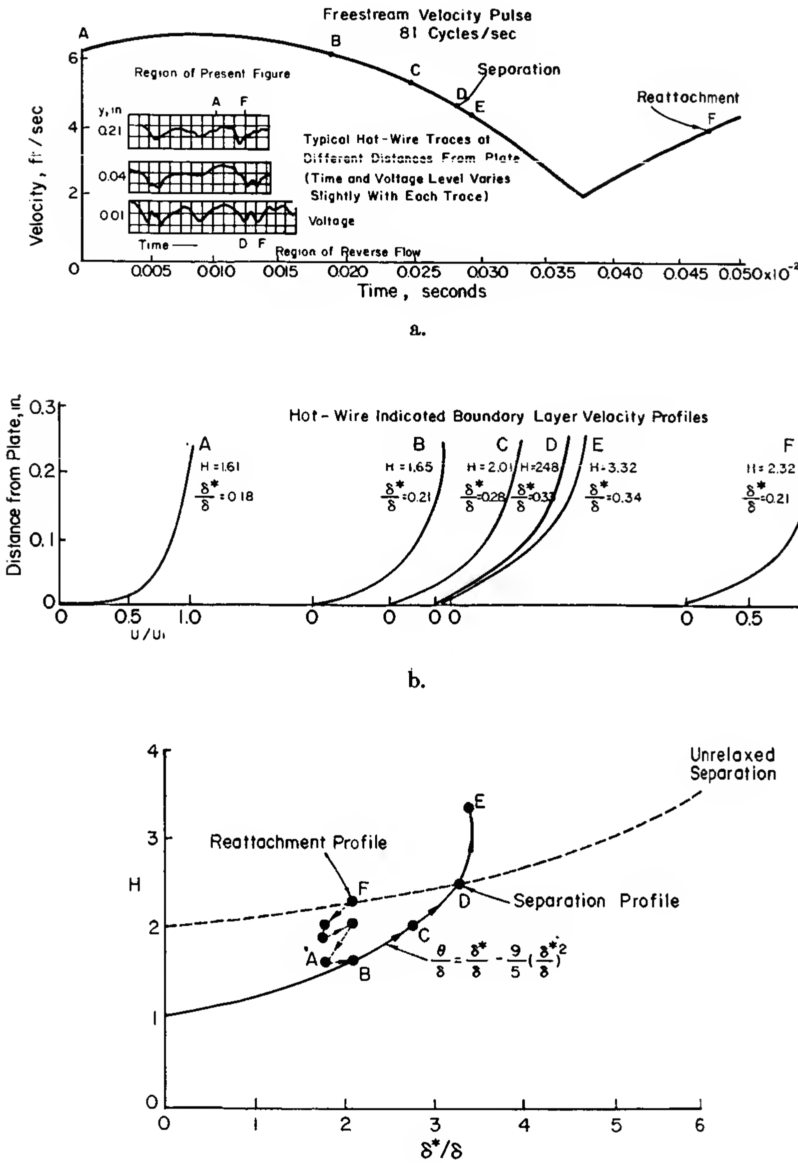
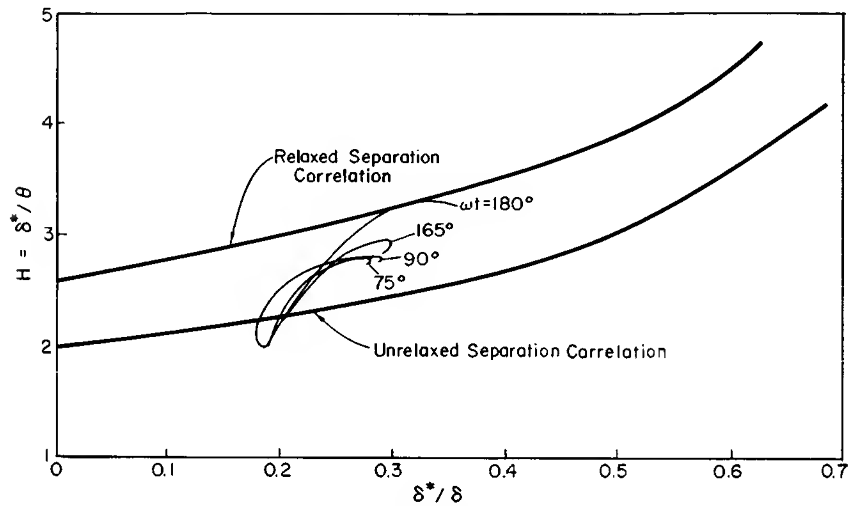


FIGURE 6.—*Turbulent boundary-layer development in a pulsing flow.*

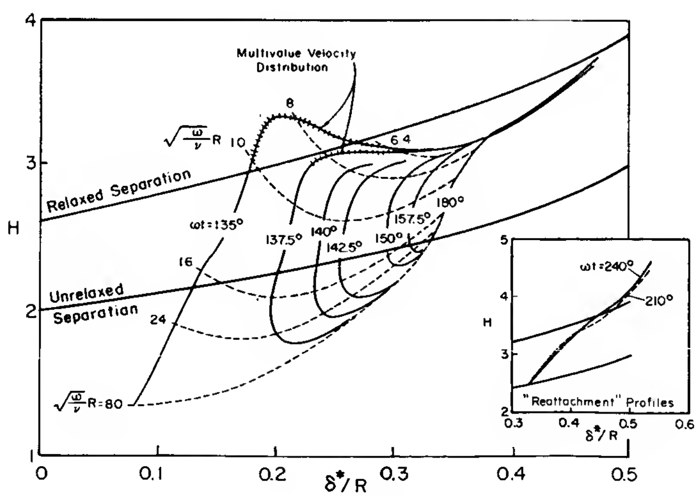
where $\zeta(t)$ is a function that may be related to the wall-shear stress. At unrelaxed separation it is required that $\zeta(t)=1$. For the constant $\frac{9}{5}$ in equation (4), it is required that $m=2$. If m is a constant, this implies that the shape of the outer region of the velocity profile is similar. Similarity

of the outer region of the velocity profile is equivalent to the requirements for an unrelaxed separation.

Recently, Chou and Sandborn (ref. 18) have evaluated the properties of two special laminar flows with time-varying free-stream velocity. Exact solutions exist for transient Poiseuille flow (ref. 19) and a special suction-type boundary layer studied by Stuart (ref. 20). Figure 7 is a



a.—*Stuart's suction boundary layer.*



b.—*Transient Poiseuille flow.*

FIGURE 7.—*Variation of transient flow separation parameters.*

plot of the velocity profile form factor, H, versus δ^*/δ or δ^*/R for families of possible separation profiles. Figure 7*a* is for the flow studied by Stuart and figure 7*b* is for the transient Poiseuille flow. The parameter ωt is a frequency-time parameter for the free-stream flow. Variation along an ωt line represents a variation in amplitude of the free-stream velocity. These curves are for "separation" cases ($\tau_w(t)=0$) in which no reversal of flow occurs. For boundary-layer consideration, it appears that velocity distributions with multivalued velocities should also be excluded. Some multivalued distributions are noted in figure 7*b*, and others have not been plotted. Curves for constant values of ωt are also included in figure 3 in terms of H versus λ_θ. Further analysis is still required to fully evaluate the limits indicated by the laminar time-varying flows. The curve for $\omega t=135°$ in figure 7*b* was the lowest value for which a reasonable separation occurred. The limit, $\sqrt{\omega/\nu}R=80$, was the largest value of the special Bessel function computed in the study. The range of possible separation velocity distributions far exceeds the unrelaxed and relaxed separation limits proposed for boundary-layer data. These theoretical solutions indicate that the assumption of a one-parameter family of profiles at the separation limits may be too restricted.

REATTACHMENT OF BOUNDARY LAYERS

Boundary-layer reattachment is found to be basically the reverse of separation. Sandborn and Liu (ref. 4) demonstrated that a number of reattachment profiles that have been reported belong to the relaxed separation family of profiles. No doubt a region should be postulated where the reattachment profile "relaxes" back to a boundary-layer profile. This relaxation in reverse has not been explored. The reattachment profile for the pulsing free-stream flow, shown in figure 6, was found to fall on the unrelaxed separation correlation curve. The insert in figure 7*b* shows parameters for reattaching velocity distributions in transient Poiseuille flow ranging over the complete region from a point which is greater than the relaxed separation value of H to the unrelaxed separation correlation curve.

An interesting evaluation of a set of laminar separation bubble and reattachment measurements reported by Gault (ref. 9) is shown in figure 8.[2] It is assumed that the separation bubble is a region detached from the main flow, so that only the flow above the bubble is considered. The "edge" of the bubble is taken where $U=0$ for $y>0$. The parameters for the velocity distributions above the bubble were computed and are

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[2] This evaluation was made by Dr. A. T. Roper in May 1965 as part of a class assignment for the author.

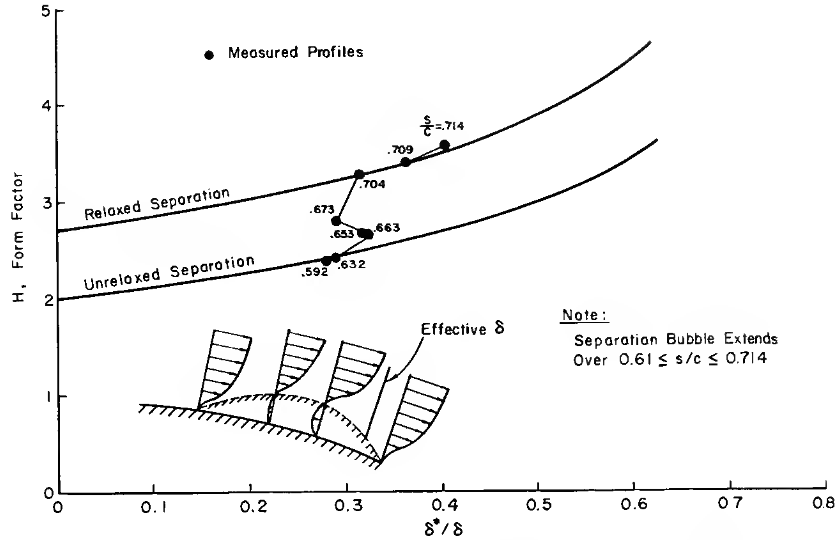

FIGURE 8.—*Profile form factor variation above a separation bubble region.*

plotted in figure 8. This particular flow was for a case where the laminar separation was of an unrelaxed type. As can be seen by the variation of the form factor, H, the velocity distribution relaxed to the relaxed separation curve. Once the profile reached the relaxed condition, it remained on the curve until reattachment occurred. This behavior of an experimental boundary layer is taken as a demonstration of the present model boundary-layer separation.

CONCLUSIONS

An improved model for boundary-layer separation has been proposed. This new model is an updating and improvement of the model proposed by Sandborn and Kline. The major new step is the introduction of a boundary-layer adjustment or relaxation time, rather than the steady and unsteady description employed in the earlier model. The relaxation time or distance concept allows the inclusion of transient boundary-layer separation in the model.

Engineering criteria for laminar and turbulent separation are presented. These criteria fit within the framework of the proposed model. However, the curves presented were determined from available experimental data and analytical solutions.

Several cases of time-dependent free-stream flows were evaluated. These flows demonstrate the concept of a relaxed model for boundary-layer

separation. The theoretical solutions for laminar time-varying free-stream flows appear to produce a more complex picture of separation than that of the proposed model.

Boundary-layer reattachment is found to follow the same model as separation. It is demonstrated that one set of laminar separation bubble data gives a graphic representation of the proposed model.

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DISCUSSION

G. SOVRAN (General Motors Research Laboratory): In spite of the universal recognition of the vital role that boundary-layer separation plays in the operation of high-performance turbomachinery, this discussor is amazed at how few investigators have the fortitude to tackle this very challenging problem. The author is to be complimented for his continuing efforts to develop understanding of this exceedingly complex fluid-mechanical phenomenon.

In this latest in a series of papers, the author introduces the concept of a boundary-layer adjustment or relaxation time into his separation model, as well as the new general definition of boundary-layer separation as "the removal of viscous restraints at the wall." I find these to be very interesting ideas, but since this is a turbomachinery symposium I would like to direct my comments in another direction. The companion papers in this session, particularly the one by Peter Bradshaw, point out the great differences that exist between the extremely complex boundary layers found in turbomachines and the much simpler two-dimensional ones that have been studied in research facilities. In addition, the turbomachine boundary layers are primarily *turbulent* in nature. Furthermore, the turbomachinery designer's game with separation is one of brinksmanship. He cannot make competitive fluid-mechanical designs if he avoids it by too great a margin; nor can he afford the consequences of inadvertently triggering it while attempting to achieve maximum performance. However, he has little interest in the details of separation once it occurs—he just wants to avoid it.

In view of these facts, and recognizing the author's own statement that "the criterion for turbulent separation given in figure 3 is at present of limited value," how would he interpret the significance of his separation model and correlations to the particular problem of turbomachinery design? How would he decide, in an a priori manner, whether a particular turbulent boundary layer would be subject to the relaxed or unrelaxed separation criterion?

Finally, the origin of the unrelaxed curves of figure 4*b* is not clear. Was the separation velocity profile corresponding to the unrelaxed *plane* boundary layers of figure 2 formally applied to axisymmetric flow geometries? If so, what about the effects of transverse wall curvature on this velocity profile?

J. M. ROBERTSON (University of Illinois): The author's contribution to our understanding of complex separation occurrences is a subject in need of considerable elucidation. The concept of the boundary layer at separation being inertia controlled and thus the separation velocity profile being essentially the same whether the layer from which it developed was laminar or turbulent seems well established. That the laminar layer developing towards separation is less likely to encompass appreciable lengths of adjustment seems rational; in fact, this writer doubts that the laminar layer will involve appreciable "unrelaxed separation." Interpretation of laminar velocity-profile measurements near separation is fraught with uncertainty due to the scarcity of measurement points in the very thin layers usually encountered, so that verification of laminar occurrences is difficult.

For the past half-dozen years, the writer has been working on the problem of calculating the boundary-layer growth on turbomachinery blading. Separation-flow considerations must be considered in the overall analysis of such flows and, in general, I have found that our understanding of these is hardly sufficient to permit adequate calculation. As the flow proceeds along the blade surface, the first question which appears is that of transition versus separation of the laminar layer which must perforce precede the more common turbulent layer, if only for a short distance. The question is one of the laminar separation occurrence and where it is likely to occur, as well as whether the subsequent flow will reattach as the "short bubble" or stay separated as "stall." The contributions of I. Tani (ref. D–1) and others (such as A. Roshko and J. C. Lau, ref. D–2) to these questions are most useful but still leave something to be desired for one attempting boundary-layer predictions. Assuming that a short bubble occurs, the next question is how the reattached turbulent layer, with its excess turbulence, relaxes back to the more-normal turbulent layer otherwise characteristic of that locale on the blade surface. Should the laminar layer simply have transitioned, rather than separated, the initial condition for the turbulent layer is merely one of equivalence of the momentum thickness at the transition "point." As the layer further develops along the blade, the possibility of turbulent separation raises its ugly head. Ultimately, no matter how the layer reaches the end of the blade (i.e., as separated or not), the final separated flow problem is one of analyzing the development of the turbulent wake, as has been noted elsewhere (ref. D–3).

The writer finds some difficulty in accepting the phenomenon of appreciable adjustment between the two separation curves for the laminar layer near separation. Such an occurrence seems to be predicated upon just two laminar layer measurements having H values slightly less than 2.5. In view of the fact that the surface pressure distribution near and ahead of the separation region for these flows was not changing rapidly, the

approach to separation can hardly be classed as rapid. It seems hard to conceive of the flow development in these cases being such that the shape factor could fall below the zero-pressure-gradient (flat-plate) value of 2.596. As may be seen in figure D–1, the laminar flat-plate occurrence appears slightly above the author's "unrelaxed separation" curve. This writer suggests that boundary-layer development from the flat-plate condition towards separation should appear simply—as without reversal in trend—on such a plot as this; thus the suggested trends are indicated in the figure for the laminar and turbulent cases from the established flat-plate locales. Also shown in this figure is the laminar separation "point" ($H=3.889$, $\delta^*/\delta=0.500$, and $\lambda_\theta=-0.0992$) given by the laminar velocity-profile formulation of B. E. Launder (ref. D–4) which, when used with the Karman momentum-integral equation, we have found much superior to other rather quick methods[D-1] of laminar-layer calculation. Separation locale prediction by this analysis has been found to agree with about 12 observations recorded in the literature at about 95 percent (with a scatter of about 5 percent) of the distance from the locale of minimum pressure on the blade surface.

If the difficult question of whether the separated laminar layer after transition as a free shear layer will reattach to form the short bubble is answered affirmatively, the analyst is then faced with the problem of how

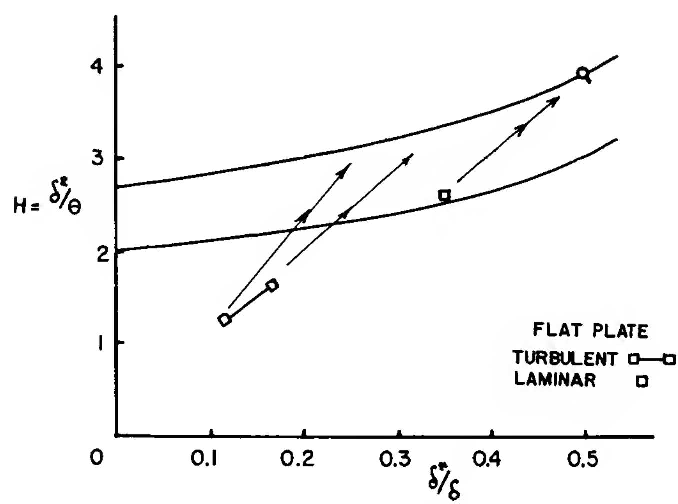

FIGURE D–1.—*Possible development of boundary layer from flat-plate condition toward separation.*

[D-1] Thus the quadrature method of Waltz, Tani, and Thwaites (1941, 1949).

the turbulent layer redevelops from its disturbed condition back into a more proper layer. After noting this problem the author gives it short shrift, presumably deferring it to later analysis. The writer is currently in the throes of developing the analysis for this occurrence. Measurements of redeveloping flow on a flat plate reported by T. J. Mueller and J. M. Robertson (ref. D–5) are quite suggestive in this regard. At reattachment, the skin friction was approximately zero and H had a value of the order of 4.0. Subsequent redevelopment of the turbulent layer towards the normal flat-plate condition extended over a distance of about 30 times the height of the bubble. However, initial changes were rapid and in 10 heights the friction factor was within 25 percent of the flat-plate value and H was less than 10-percent larger than the corresponding flat-plate value. The early decay in H from the high reattachment value was roughly as the logarithm of the distance from reattachment. From this decay rate and appropriate formulations for C_f in terms of the momentum-thickness Reynolds number and H, it seems possible to predict the increase in C_f for redeveloping flows. Then boundary-layer development prediction via the momentum integral equation follows simply. Verification studies of this approach are underway.

SANDBORN (author): The author would like to thank the reviewers for their comments. The reviewers point out the extreme difficulty that still exists in an adequate engineering prediction of boundary-layer separation. As noted by the reviewers, the present separation model is still limited in its engineering application. The present discussion is intended mainly as a starting point from which engineering analysis can proceed. The model presented suggests a "one parameter" pressure-gradient type of engineering criterion for both laminar and turbulent separation. Unfortunately, methods of calculating turbulent boundary layers up to separation are not sufficiently accurate to make the turbulent separation criterion usable. Improvements in boundary-layer calculation techniques are necessary in order that the turbulent separation criterion (fig. 3) can be of engineering value.

In answer to Dr. Sovran's question about a turbulent boundary layer being subject to the relaxed or unrelaxed separation criterion, it appears that the turbulent boundary layer always separates along the unrelaxed curve. For turbulent separation the relaxed separation criterion may not be of major engineering importance, since the unwanted effects of separation are present at the unrelaxed separation point.

The axisymmetric separation correlation curves shown in figure 4 were obtained by applying the definitions of equation (3) to the empirical velocity profile of Sandborn (ref. 17). The agreement of experimental measurements with the curves of figure 4*b* suggests that the transverse wall curvature does not produce a major effect on the integral boundary-layer parameter.

Dr. Robertson has pointed out the difficulties involved if laminar separation bubbles are encountered. The present model for separation is unable to produce information on whether the laminar flow will reattach. It is pointed out by Dr. Robertson that a reattaching boundary layer will, so to speak, "unadjust" to the turbulent boundary layer again. This suggests that the words used in the present model may still be subject to change.

For most laminar flows, one may suspect that separation occurs along the relaxed correlation curve. However, the two cases shown in figure 2 are definitely exceptions to the rule. For these two cases, the free-stream turbulence level had been increased. The point raised by Dr. Robertson that unrelaxed separation is not necessarily related to a rapid approach to separation may well be valid. It would appear that free-stream turbulence level as well as pressure gradient can be a factor for laminar separation. Certainly, for the time-dependent free-stream flows, the concept of a rapid change in pressure gradient can produce the unrelaxed types of laminar separation.

A recent look at turbulent separation profiles suggests that the major relaxation process in the region from unrelaxed to relaxed separation takes place near the wall. The outer profile is nearly wakelike at the point of unrelaxed separation, and the inner flow adjusts to the zero wall-shear stress condition over the transition region. This relaxation process appears to be just the reverse of that observed for time-dependent free-stream flows.

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